

Section P.3 Functions and Their Graphs

Ex.1 Evaluate the function and simplify:  $f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases}$ ,

find (a)  $f(-2)$ , (b)  $f(0)$ , (c)  $f(1)$  and (d)  $f(s^2 + 1)$

$$\begin{aligned} \text{(a)} \quad f(-2) &= (-2)^2 + 2 \\ &= 4 + 2 \\ f(-2) &= 6 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad f(0) &= (0)^2 + 2 \\ &= 0 + 2 \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad f(1) &= (1)^2 + 2 \\ &= 1 + 2 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad f(s^2 + 1) &= 2(s^2 + 1) + 2 \\ &= 2s^2 + 2 + 2 \\ &= 2s^2 + 4 \end{aligned}$$

Ex.2 Evaluate the function and simplify:  $f(x) = x^3 - x$ , find  $\frac{f(x)-1}{x-1}$

$$\begin{aligned} \frac{f(x)-1}{x-1} &= \frac{(x^3 - x) - 1}{x-1} \\ &= \frac{x^3 - x - 1}{x-1} \end{aligned}$$

Ex.3 Find the following functions:

(a)  $f(x) + g(x)$ , (b)  $f(x) - g(x)$ , (c)  $f(x) \cdot g(x)$  and (d)  $f(x)/g(x)$ ,

given:  $f(x) = x^2 + 5x + 4$  and  $g(x) = x + 1$

$$\begin{aligned} \text{(a) } f(x) + g(x) &= (x^2 + 5x + 4) + (x + 1) \\ &= x^2 + 6x + 5 \end{aligned}$$

$$\begin{aligned} \text{(b) } f(x) - g(x) &= (x^2 + 5x + 4) - (x + 1) \\ &= x^2 + 5x + 4 - x - 1 \\ &= x^2 + 4x + 3 \end{aligned}$$

Ex.3

$$(c) f(x) \cdot g(x) = [x^2 + 5x + 4] \cdot [x + 1]$$

$$= (x^2)(x) + (x^2)(1) + (5x)(x) + (5x)(1) + (4)(x) + (4)(1)$$

$$= x^3 + x^2 + 5x^2 + 5x + 4x + 4$$

$$= x^3 + 6x^2 + 9x + 4$$

$$(d) f(x)/g(x) = \frac{(x^2 + 5x + 4)}{(x+1)}$$

$$= \frac{(x+1)(x+4)}{(x+1)}$$

$$= x+4$$

Ex.4 Find the following functions: (a)  $(f \circ g)(x)$  and (b)  $(g \circ f)(x)$ ,

given:  $f(x) = \frac{x}{x-1}$  and  $g(x) = \frac{-4}{x}$

$$(a) (f \circ g)(x) = f(g(x))$$

$$= \frac{g(x)}{g(x) - 1}$$

$$= \frac{\frac{-4}{x}}{\frac{-4}{x} - \frac{x}{1}}$$

$$= \frac{\frac{-4}{x}}{\frac{-4}{x} - 1} = \frac{\left[ \frac{-4}{x} \right]}{\left[ \frac{-4}{x} - 1 \right]} \cdot \left( \frac{\frac{x}{1}}{\frac{x}{1}} \right)$$
$$= \frac{-4}{-4 - x} = \frac{-1 \cdot (4)}{-1 \cdot (x+4)} = \frac{4}{x+4} \checkmark$$

$$(b) (g \circ f)(x),$$

$$= g(f(x))$$

$$= \frac{-4}{f(x)}$$

$$= \frac{-4}{\frac{x}{x-1}}$$

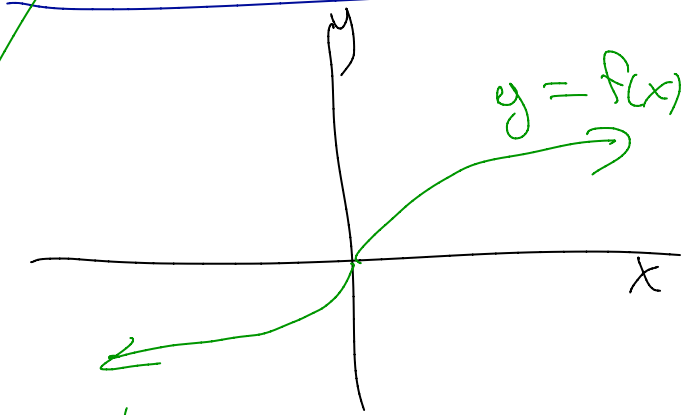
$$= \frac{-4}{1} \cdot \frac{x-1}{x}$$
$$= \frac{-4x+4}{x} \checkmark$$

Ex.5 determine whether the following functions are even, odd, or neither:

(a)  $f(x) = \sqrt[3]{x}$  and (b)  $g(x) = \sin^2(x)$

$$\begin{aligned} \text{(a)} \quad f(-x) &= \sqrt[3]{(-x)} \\ f(-x) &= -\sqrt[3]{x} \\ f(-x) &= -f(x) \end{aligned}$$

$f$  is odd



Symmetric  
About the  
origin

$$\text{(b)} \quad g(-x) = \sin^2(-x)$$

$$g(-x) = [\sin(-x)]^2$$

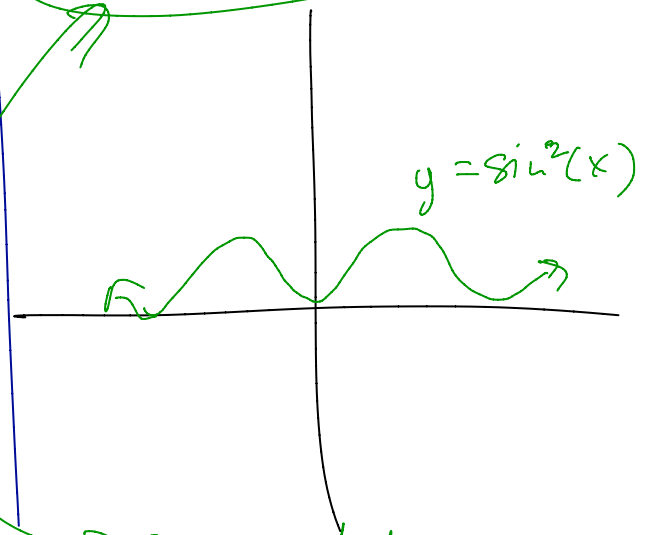
$$g(-x) = [-\sin(x)]^2$$

↑  
since  
the sine  
function is odd

$$g(-x) = \sin^2(x)$$

$$g(-x) = g(x)$$

$g$  is even



Symmetric  
About the  
y-axis